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Experimental study on the statistics of wall shear stress in turbulent channel flows

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Measurements of local wall shear stress have been undertaken using a laser gradient meter that enables direct evaluation of the wall velocity gradient with high spatial resolution. Mean values as well as the statistical parameters of fluctuating wall shear stress, such as third and fourth moments, have been obtained in fully developed air flow in a two-dimensional channel at moderate Reynolds numbers. Agreement with the available direct numerical simulation (DNS) and experimental results is satisfactory. The dependence of the turbulence statistics on the Reynolds number is masked by the experimental ambiguity. Although the linear near-wall velocity profile is the principal requirement for the application of the present system, it is found that the measurement of the mean velocity gradient is less influenced by the nonlinearity of the velocity profile more distant from the wall than are the turbulence statistics.

Keywords: local wall shear stress; two-dimensional channel flow; turbulence statistics; optical measurement; dual cylindrical wave

Introduction

Detailed structure of the near-wall turbulence has been intensively investigated through a number of direct numerical simulations (DNS). For instance, the relationships between the bursting phenomena, coherent motion in the inner layer, and intermittent characteristics of fluctuating wall shear stress have been analyzed in detail (Kim et al. 1987). As far as simple boundary-layer flows are concerned, measurements by conventional techniques such as using glue-on hot-film sensors provide fairly good agreement with results of DNS, when special care is taken to prevent heat loss due to conduction by the wall (Alfredsson et al. 1988). However, it is obvious that investigation of more complex problems comprising separation and reattachment of the turbulent boundary layer will fall within the scope of DNS, when the rapid development of computer technique is considered, and more advanced experimental techniques will be required to provide accurate reference data for such complex flows.

Although there are a number of techniques for measuring the wall shear stress, methods which are suitable for measuring the fluctuating local wall shear stress are rather limited (Haritonidis 1989). The instantaneous shear stress would be most directly obtained by measuring shear force acting on a small portion of the wall. However, due to the finite mass of the sensing float, the frequency response of this type of direct method is not satisfactory, or a sensor with very high sensitivity is required, which is difficult to handle. The hot-film technique and its variations have been widely used for the detailed investigation of fluctuating wall shear stress. This type of indirect method requires calibrations; therefore, the application is limited to relatively simple flows, and the measurements in "unknown" flows will be erroneous. For example, the analogy between the heat transfer and wall shear stress, i.e., the basis of the hot-film measurements, is violated in separated flows. The near-wall velocity measurement by a direction-sensitive laser-Doppler anemometer (LDA) is a candidate for evaluating both magnitude and direction of the wall shear stress. However, it is usually very difficult to determine the path of tracers unambiguously, which is required for evaluating the wall velocity gradient.

Recently, an optical method capable of measuring the wall shear stress with high spatial resolution was proposed by Naqwi and Reynolds (1991). Instead of the control volume with parallel fringes as in the case of LDA, its control volume possesses a fanlike fringe pattern adjacent to the wall, so that the velocity gradient at the wall is directly evaluated from the light scattered by the tracer particles. When used with a frequency-shifting device, it enables the detection of reverse flow as well. However, difficulties in hardware implementation, such as the method of seeding to maintain a sufficiently high data rate without contaminating the wall, as well as of data acquisition in the region very close to the wall, have hindered its wide application, particularly to gas flow measurements.

The objective of the present study is assessment of this method through the measurement of fully developed turbulent channel flow, for which a sufficient number of experiments and DNS have already been performed. The difficulty of data acquisition is alleviated by adopting an advanced data-processing technique in place of using a conventional counter-type signal processor. The results are presented for the higher-order moments of fluctuating local wall shear stress at various Reynolds numbers, and discussions will be given on the feasibility and further applicability of the technique.

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Principle

The method is here called the laser gradient meter (LGM) method, for convenience. Its principle may be explained using a fanlike optical fringe pattern, as shown in Figure 1. Here, the light source is a pair of closely spaced spanwise optical slits, and the diffraction of laser beams projected from behind the wall results in cylindrical waves which propagate radially from each slit. Young's theory applied to the wave interference shows that a fanlike fringe pattern is formed, originating from the wall surface, with the fringe spacing d being approximately proportional to the distance from the wall,

$$d \approx \frac{\lambda y}{S} \tag{1}$$

where S and λ denote the slit spacing and the wavelength of the light, respectively.

Tracer particles suspended in the flow, that pass across the fringes, scatter the light containing the shift frequency, just as they do for an LDA. The "Doppler-shift" frequency is proportional to the velocity of the tracer particles and inversely proportional to the local fringe spacing at the tracer path,

$$f = \frac{u(y)}{d(y)} \tag{2}$$

On the other hand, the instantaneous velocity component parallel to the wall may be expressed in the vicinity of the wall as

$$u = ay + O(y^2) \tag{3}$$

Here, the coefficient a corresponds to the velocity gradient at $y = 0$. The higher-order term on the right-hand side may be neglected in the absence of an appreciable effect of the streamwise pressure gradient.

From the above equations, we obtain the following relationship which indicates that the frequency of scattered light f is proportional to the velocity gradient a,

$$f = \frac{S}{\lambda} a \tag{4}$$

The direct measurement of instantaneous velocity gradient a is therefore possible by detecting the frequency f, without the knowledge of the location of the tracer path. The instantaneous wall shear stress $\hat{\tau}_w$ can thus be evaluated as $\hat{\tau}_w = \mu a$, with μ being the fluid viscosity. The advantages of this method are that extreme precision in determining the probe location, such as is the case with, e.g., the hot-wire anemometer, is unnecessary, and that the scattered light can be detected by the conventional receiving optics of an LDA.

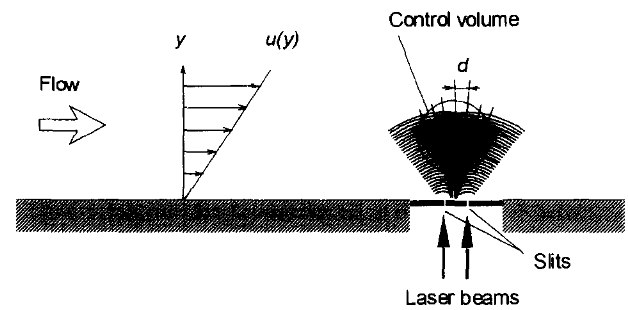

Figure 1 Control volume of LGM

Instrumentation

The transmitting optics consisted of a 2W Ar-ion laser tube ($\lambda = 514.5$ nm), beam splitter, frequency shifter, beam integrator, and cylindrical lens, as shown in Figure 2. The laser beam was first split by a polarization-preserving prism into two beams with equal intensity. Each beam was led to a Bragg cell module by which the frequency was shifted. The beams were integrated by a partial mirror, but not aligned, and then focused by a cylindrical lens of 5 mm focal length onto the slits. To project each beam

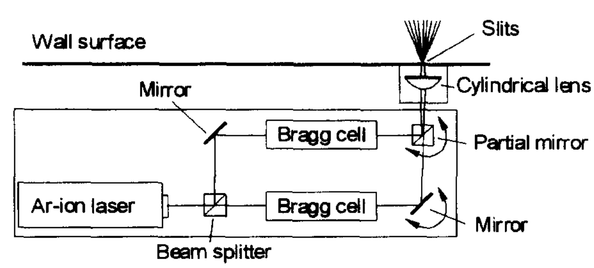

Figure 2 Arrangement of transmitting optics

Notation

a	velocity gradient at the wall
C_f	skin friction coefficient $C_f = \frac{\tau_w}{\frac{1}{2}\rho U_C^2}$
d	fringe spacing
dp/dx	streamwise pressure gradient
f	frequency of the scattered light
H	channel height
$P(\hat{\tau}_w)$	probability density of wall shear stress
S	slit spacing
U^+	streamwise mean velocity normalized by friction velocity
U_c	channel center velocity
u	streamwise velocity
u^+	fluctuating component of streamwise velocity normalized by friction velocity
y	wall-normal coordinate
y^+	wall-normal coordinate normalized by wall unit
y_0	the closest location to the wall at which a significant burst signal is detected

Greek

λ	wavelength of laser light
μ	fluid viscosity
ρ	fluid density
τ_w	wall shear stress
$\hat{\tau}_w$	instantaneous wall shear stress
τ'_w	root-mean-square of the fluctuating wall shear stress
τ^*_w	reduced wall shear stress, $\tau^*_w = (\hat{\tau}_w - \tau_w)/\tau'_w$
τ_{w0}	τ_w at $y = y_0$
τ'_{w0}	τ'_w at $y = y_0$

separately onto the individual slits, the angle of incidence to the cylindrical lens was manipulated by adjusting the tilt of mirrors. Because the cylindrical lens must be precisely aligned with respect to the slits, it was fixed onto the channel wall together with the slit assembly.

The slits were etched on an approximately 1-μm thick chromium coating on an optical glass plate of 2-mm thickness by means of electron-beam lithography. The slits were 1-μm wide and 400-μm long, and the distance between the slit centerlines was 11 μm. The divergence angle of the resulting fan fringe was calculated to be 2.95°, and the fringe pattern was formed in the region $y \geq 8.33$ μm.

The scattered light was collected in the side-scatter mode. The receiving optics consisted of a collecting lens of 40 mm in diameter with 240-μm focal length, combined with a spatial filter comprising a set of collimator lenses and a pinhole of 25-μm diameter so that the control volume diameter was reduced to 100 μm. All of these components were mounted in a precisely machined brass sleeve, and the detected light was transmitted through a multimode optical fiber to a photomultiplier assembly.

A signal processor based on the fast Fourier transform (FFT) burst analyzer (Maeda et al. 1989) was used for the evaluation of "Doppler-burst" frequency. The whole system was integrated on a DSP board mounted in a 16-bit personal computer. The frequency of the burst signal was typically on the order of 100 kHz. The data rate ranged between 2 and 10 Hz, depending on the measurement conditions. The statistical parameters were calculated from 8000 to 10,000 samples.

Flow system

Figure 3 illustrates the setup for the channel flow measurements. The two-dimensional (2-D) channel flow was obtained in a suction duct 20 mm high and 2 m long. The channel aspect ratio was 1:32. The whole channel was made of aluminum plates 10 mm thick, and both walls were rimmed every 150 mm in the streamwise direction to avoid bending due to the pressure difference between inside and outside the channel. From preliminary velocity measurements using a hot-wire anemometer, the spanwise uniformity of the channel center velocity was confirmed to be within ±1% of the averaged value over at least 20 channel heights across the channel span.

The oncoming flow was tripped using #80 sandpaper strips, 15 mm wide, glued across the full span on both walls of the channel at the inlet. The development of channel flow was examined from the streamwise wall pressure variation measured along the channel centerline. The linearity of the streamwise pressure gradient around the measuring position was confirmed to be within 0.5% of the average dp/dx.

The measuring position for the local wall shear stress was located 1600 mm from the channel inlet, where an aluminum plug with the optical slit assembly was mounted flush with the surface. The channel wall on the opposite side of the plug was replaced by a transparent Plexiglass plate of 100×60 mm to provide a path for scattered light. The receiving optics was fixed on a 3-D traversing table, the offset angle being about 30° from the wall.

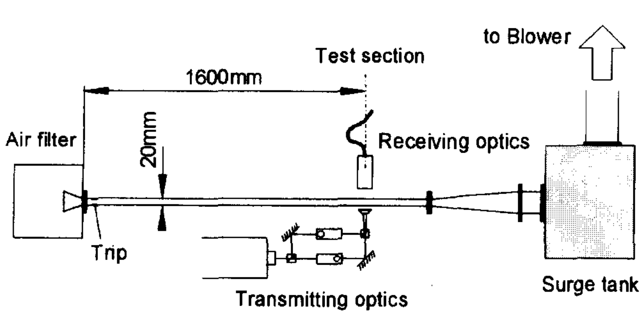

Figure 3 Experimental setup for channel flow measurements

Water mist from an ultrasonic atomizer was carefully injected 320-mm upstream of the test section through a spanwise slit 2 mm wide and 40 mm long located in the test wall. The mean diameter of water mist particles was estimated to be on the order of 1 μm. The humidity in the laboratory was low enough to prevent the test section's being dim.

It should be noted that the apparently low datarate mentioned in the previous section was attributable to several reasons, including the unsatisfactory number of tracer particles entering the control volume. However, the main reason was the optical noise due to the laser light scattering at the slit edge, which seriously injured the signal quality. This influence could be minimized by orienting the receiving optics nearly parallel to the wall, but, in the present test section, it was impossible to reduce the receiving angle, because the signal was collected through the opposite side of the channel. Nevertheless, the position and the orientation of the receiving optics were optimized to maximize the data rate, and the resulting value was found satisfactory for the further process.

Results

Preliminary assessment

From our previous investigations, it has been confirmed that the present equipment is applicable to a viscous sublayer thicker than 150 μm (Inoue et al. 1994). Assuming the edge of the viscous sublayer to be $y^+ = 5$, where y^+ is nondimensional wall distance based on the wall unit, and using the empirical relationship of Dean (1978), this limit can be converted to the corresponding Reynolds number range of $\text{Re} \leq 1.3 \times 10^4$. Here, Reynolds number Re is based on the channel width $2H$ (20 mm) and the channel center velocity U_C. Among a number of channel flow DNS studies, we choose the work done by Kim et al. (1987) as a reference because of the moderate Reynolds number (Re = 6600) and the completeness in terms of the information of turbulence statistical parameters.

To examine the velocity field developing in the channel, the distributions of mean and fluctuating velocity components are measured with a hot-wire anemometer employing an I-probe, and results are compared with the DNS results of Kim et al. (1987) in Figure 4. The mean and fluctuating velocities as well as the wall distance are normalized by the friction velocity obtained by the LGM measurement described later. Except for the inevitable deviation in the vicinity of the wall due to the wall proximity effect on the hot-wire anemometer, agreement with the DNS data for both mean and fluctuating velocity components is satisfactory for a subsequent test of LGM.

Time-averaged wall shear stress

In Figure 5, the mean velocity gradient, as evaluated from the LGM measurement, is compared with the near-wall mean velocity profile from the hot-wire data. The hot-wire data close to the wall are modified according to Bathia et al. (1982). It is demonstrated that the solid line representing the LGM result agrees very well with the corrected hot-wire data in the viscous sublayer for $y^+ \leq 5$. As already indicated in Figure 4, consistency of the normalized velocity profile has also been achieved by using the friction velocity based on this wall velocity gradient.

The control volume of LGM is schematically shown in Figure 5. It is confirmed that the viscous sublayer is, indeed, thick

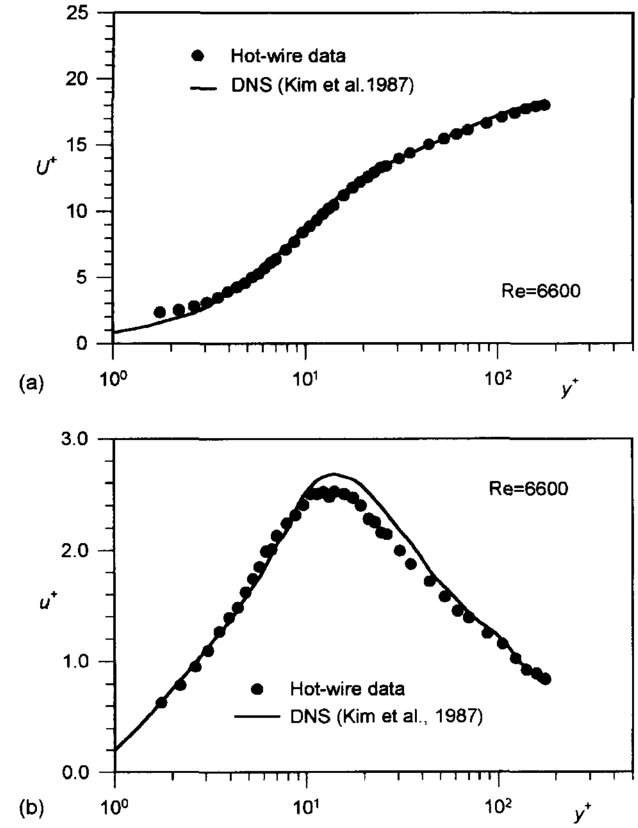

Figure 4 Preliminary velocity measurements in the channel flow and comparison with DNS: (a) streamwise mean velocity; (b) streamwise velocity fluctuation

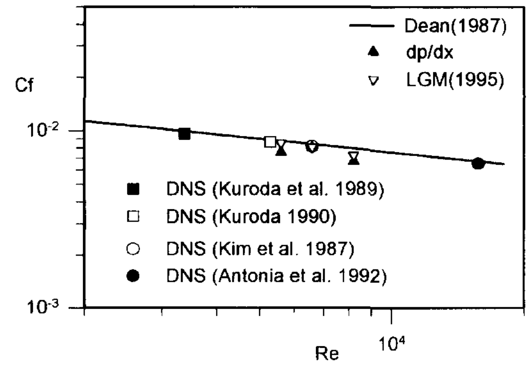

Figure 6 Effect of Reynolds number on the skin friction coefficient

enough to cover the entire control volume. Note that the absolute position of the control volume is not known. The influence of the ambiguity in the control volume position is discussed later.

Figure 6 summarizes the mean wall shear stress in the form of the skin friction coefficient normalized by the dynamic head based on U_C, and compares the result of the LGM and those evaluated from the pressure gradient in the present experiment, by DNS and using Dean's (1978) empirical formula. The obtained variation of skin friction coefficient against the Reynolds number is in good agreement with the DNS results.

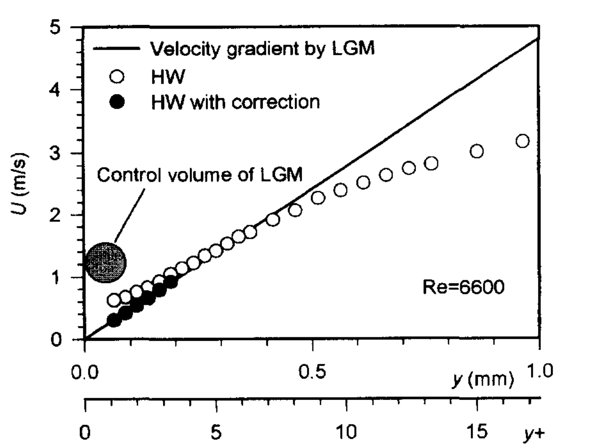

Figure 5 Comparison of velocity gradient measured by LGM and near-wall velocity profile from hot-wire measurement

Probability density function of wall shear stress

The probability density function (PDF) of the fluctuating wall shear stress is presented in Figure 7 for Re = 6600. The result obtained by the LGM is compared with those from DNS by Kim et al. (1987). (The PDF is not presented in their original paper. The solid line presented here is taken from Naqwi and Reynolds 1991.) The variable τ_w^* on the abscissa is defined as the deviation of the instantaneous wall shear stress from the mean value normalized by the rms value τ_w', i.e., $\tau_w^* = (\hat{\tau}_w - \tau_w)/\tau_w'$. The PDF is normalized so that the integrated value becomes unity. It is found that the distribution is positively skewed due to the common intermittent feature of the viscous sublayer. The agreement with the DNS results is satisfactory.

Turbulence statistics

The statistical parameters of the fluctuating wall shear stress obtained in the present study are summarized in Table 1. The results of other studies are also presented, including those for channel flows, a pipe flow, and boundary-layer flows. The results of other experiments in channel flows are scattered between 0.06 and 0.40 for the relative level of the shear stress fluctuation, between 0.84 and 1.1 for the skewness factor (SF) and between 4.1 and 4.8 for the flatness factor (FF). The present results fall within these scatter ranges for all these quantities.

The experiment of Chambers et al. (1983) is the only single instance of one, apart from the present case, which was carried out in air. As pointed out by Alfredsson et al. (1988), the use of hot film in air causes more error due to heat conduction into the wall compared with liquid flow cases. The excessively low value of τ_w'/τ_w obtained by Chambers et al. is probably attributable to

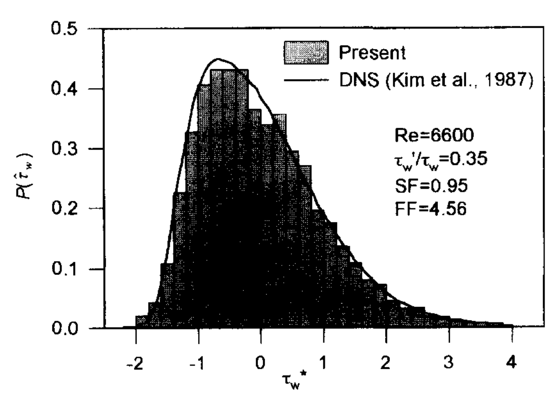

Figure 7 Probability density function of wall shear stress

Table 1 Statistical parameters of wall shear stress in simple wall flows

Authors	Flow type	Re	Technique	Medium	τ_w'/τ_w	Skewness factor	Flatness factor
Present	Channel	5,600	LGM	Air	0.41	1.05	4.7
	Channel	6,600	LGM	Air	0.35	0.95	4.6
Alfredsson et al. (1988)	Channel	10,000	Hot film	Water	0.40	1.0	4.2
	Channel	7,600	Hot film on wall	Oil	0.36	1.1	4.8
Chambers et al. (1983)	Channel	7,200 ~ 47,200	Hot film	Air	0.03 ~ 0.06	—	—
Eckelmann (1974)	Channel	5,600	Hot film	Oil	0.24	—	—
Kim et al. (1987)	Channel	6,600	DNS	—	0.36	0.84	4.1
Kuroda (1990)	Channel	4,560	DNS	—	0.38	0.91	4.2
Durst et al. (1993)	Pipe	7,500	LDA	Oil	0.37	0.85	4.1
Alfredsson et al. (1988)	BL	28,000*	Hot wire on wall	Air	0.39	1.0	4.8
Castro et al. (1985)	BL	815 ~ 1,202	Pulsed HW	Air	0.40	0.66 ~ 0.73	—
Dengel et al. (1987)	BL		Pulsed HW	Air	0.37	—	—
Karlsson and Johansson (1986)	BL	2,420	LDA	Water	0.40	0.9	4.0
Naqwi and Reynolds (1991)	BL	3,700	LGM	Water	0.38	0.78	4.0

*based on boundary-layer thickness; other Reynolds numbers for boundary-layer experiments are based on momentum thickness

the unsatisfactory response of the hot film to high-frequency fluctuation in air flow measurement.

The present measurement (Re = 6600) has been performed in the same Reynolds number range as in the experiment of Alfredsson et al. (1988) (Re = 7600) and DNS by Kim et al. (1987) (Re = 6600). The comparison of these three datums indicates that the agreement in τ_w'/τ_w is achieved to within 2% accuracy, while a larger scatter is found in the higher-order moments. Although the present data taken at Re = 5600 indicate a slightly higher value of τ_w'/τ_w than the others, SF and FF are smaller than in the experiment of Alfredsson et al. As a whole, the relationship between these turbulence statistics and Reynolds number seems to be so weak that it falls within the experimental uncertainty.

The experiments for boundary-layer flows provide narrower scatter of the fluctuating shear stress as compared with the channel flow experiments, regardless of the medium and measuring technique. The measurement in pipe flow by Durst et al. (1993) yielded values similar to those in the channel and boundary-layer experiments for all the quantities presented here.

From the above comparisons, the turbulence characteristics of the fluctuating wall shear stress are found to be universal, at least at the present level of uncertainty, although the mean flow structures of these three flow types are different.

A comparison in terms of the measuring technique reveals that the results obtained using optical methods by Durst et al. (LDA) (1993), Karlsson and Johansson (LDA) (1986), and Naqwi and Reynolds (LGM) (1991) are similar, with the rms value falling in the range between 0.37 and 0.40, the skewness factor between 0.78 and 0.90, and the flatness factor between 4.0 and 4.1. On the other hand, most of those values obtained by the hot-film method and its variants exhibit wider scatter ranges; i.e., $0.36 \le \tau_w'/\tau_w \le 0.40$, $0.66 \le SF \le 1.1$ and $4.2 \le FF \le 4.8$. The reason for the narrower scatter ranges in the optical methods might be the invalid data discrimination procedure that is commonly undertaken in signal processors of LDA. Because the relative turbulence level of fluctuating wall shear stress is very high and the PDF is strongly skewed, a discrimination algorithm which assumes the Gaussian PDF distribution would result in a rejection of valid data. For a proper assessment of statistical data, the time history of the fluctuating signal is necessary. A combination of the optical and hot-film techniques would provide more accurate turbulence statistics.

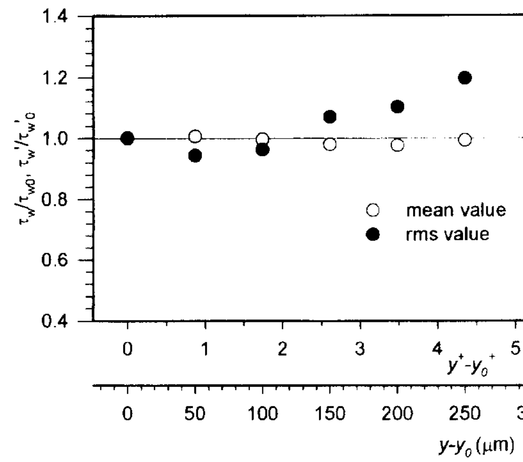

Figure 8 Effect of control volume position on the measured wall shear stress

Discussion

Naqwi and Reynolds (1991) performed an extensive analysis of the possible sources of ambiguity in the LGM measurement. They concluded that the nonlinearity in the near-wall velocity profile is the major cause of uncertainty among several factors including the variation of fringe spacing in the streamwise direction and the influence of the normal velocity component. In the present study, the influence of the nonlinear velocity distribution is examined by shifting the control volume away from the wall by moving the receiving optics, and observing the change of recorded wall shear stress.

The results of such examination are presented in Figure 8, in which the variations of the mean and rms values of the wall shear stress are plotted against the location of the control volume. The distance from the wall is normalized by the wall unit, with the origin set at the closest location to the wall at which a significant burst signal was detected. Although no information is available about the absolute position, it is presumed that $y = y_0$ roughly corresponds to the diameter of the control volume, i.e., 100 μm. Both mean and rms values of the measured wall shear stress are normalized by the values at $y = y_0$, and it is presumed that both should remain constant as long as the entire control volume is submerged in the viscous sublayer.

It is noteworthy that the mean shear stress remains approximately constant as the control volume is moved 250 μm away, which corresponds to about 4.5 wall units. The rms value is roughly constant up to $y - y_0 = 100$ μm, then monotonically increases; the apparent decrease of the rms value in the range 0 μm $\leq (y - y_0) \leq 50$ μm is perhaps attributable to the marginal signal quality at the first measuring point. The assessment of the measured wall shear stress is, therefore, possible by inspecting the variation of both the mean and the fluctuating component against the position of control volume, even if the near-wall values are of lower quality.

The above results indicate the wide applicability of the LGM; i.e., the assessment of the wall shear stress measured in any wall-bounded flow is possible by examining the wall asymptotic behavior. For example, boundary-layer flow under the influence of a strong streamwise pressure gradient does not contain the linear velocity region even in the viscous sublayer. However, once the converging behavior is established as a function of the detector position, the ambiguity in the results can be estimated. Although the operation of the LGM is restricted to use in a transparent fluid with a well-prepared optical path for the detector, it has been shown to be promising as a standard tool for local wall shear stress measurement in gas flow.

Conclusion

The LGM has been successfully applied to the measurement of wall shear stress of turbulent air flows in a two-dimensional channel at moderate Reynolds numbers. The mean values are in good agreement with those obtained from the pressure measurements as well as those of other studies, including DNS and experiments.

The standard deviation of the fluctuating wall shear stress is around 40% of the mean value. The third and fourth moments have been evaluated as about 1.0 and 4.5, respectively. The dependence of these values on the channel Reynolds number is found to be weak and cannot be evaluated with the present experimental uncertainty.

From the systematic examination of the influence of the control volume location on the results, it has been found that the mean wall shear stress is less affected by the nonlinear effect of the near-wall velocity profiles as compared with its fluctuating component. The examination of the wall asymptotic behavior indicated the possibility of measurement in more complex flows in which the linearity of the near-wall velocity is not guaranteed.

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